1 Memorandum for MATLAB program of *Credit Cycles*

1.1 Model overview; Kiyotaki and Moore (1997)

A linear production technology for entrepreneurs,

$$y_t = (a + c)\, k_{t-1}$$

A fraction ck_{t-1} of the produced goods is untradable so that the entrepreneur must consume by their own. Consider a credit constraint,

$$Rb_t = q_{t+1} k_t$$

Basic expression for the flow of fund,

$$q_t\,(k_t - k_{t-1}) + \phi\,(k_t - \lambda k_{t-1}) + Rb_{t-1} + x_t - ck_{t-1} = ak_{t-1} + b_t$$

where $\phi\,(k_t - \lambda k_{t-1})$ denotes an input for reproduction of capital. For a fraction π of the population can invest (entrepreneurs), while $(1-\pi)$ (households) cannot. For a firm, investment is strictly better than consumption so that $x_t \geq ck_t$ and $Rb_t = q_{t+1}k_t$ are binding.

$$\left(q_t + \phi - \frac{q_{t+1}}{R}\right) k_t = (a + \lambda\phi + q_t)\, k_{t-1} - Rb_{t-1}$$

On the other hand, households' capital is just depreciating, namely, $k'_t = \lambda k'_{t-1}$. Combining these two to get aggregate capital law of motion,

$$\begin{aligned} K_{t+1} &= (1-\pi)\,\lambda K_{t-1} \\ &\quad + \left(\frac{\pi}{q_t + \phi - q_{t+1}/R}\right) \left[(a + \lambda\phi + q_t)\, K_{t-1} - RB_{t-1}\right] \end{aligned} \tag{1}$$

Further debt follows,

$$B_t = RB_{t-1} + q_t\,(K_t - K_{t-1}) + \phi\,(K_t - \lambda K_{t-1}) - aK_{t-1} \tag{2}$$

Finally, the Euler equation of consumption or asset price path,

$$\frac{G'\,(K - K_t) + q_{t+1}}{q_t} = R \tag{3}$$

or in an alternative form $\psi\,(K_t) = q_t - q_{t+1}/R$, where $\psi\,(K_t) = G'\,(K - K_t)\,/R$.

1.2 Simulation on MATLAB

1.2.1 Steady state

By eqn (2),

$$\frac{B^*}{K^*} = \frac{a + \lambda\phi - \phi}{R - 1}$$

By eqn (1),

$$
\begin{aligned}
q^* \frac{R-1}{R} &= \frac{\pi a + \{\pi\lambda - (1-\lambda)(R-1) + (R-1)\pi\lambda - \pi R\}\phi}{(1-\lambda+\pi\lambda)(R-1) - \pi R} \\
&= \frac{\pi a - \phi(1-\lambda)(1-R+\pi R)}{\pi\lambda + (1-\lambda)(1-R+\pi R)}
\end{aligned}
$$

$$
\begin{aligned}
K^* &= q^* \frac{R-1}{R} + v \\
B^* &= (B/K)^* * K^*
\end{aligned}
$$

1.2.2 Dynamics

From eqn(1), (2) and (3) the system pooled in *optcon* part will be,

$$
\begin{aligned}
\mathit{Capital} &= \left(\frac{\pi}{q_t + \phi - q_{t+1}/R}\right)\left[(a + \lambda\phi + q_t) K_{t-1} - RB_{t-1}\right] + (1-\pi)\lambda K_{t-1} - K_t \\
\mathit{Debt} &= RB_{t-1} + q_t (K_t - K_{t-1}) + \phi (K_t - \lambda K_{t-1}) - aK_{t-1} - B_t \\
\mathit{Euler} &= q_t - \frac{q_{t+1}}{R} - \psi(K_t)
\end{aligned}
$$

Let $\psi(K_t) = K_t - v$, $v = 2$, $\phi = 20$, $a = 1$, $\lambda = 0.975$, $\pi = 0.1$, and $R = 1.01$. Note that the system has two stable roots and one unstable root so that the Blanchard-Kahn theorem should be satisfied.

References

[1] Kiyotaki, N. and J. Moore (1997), "Credit Cycles" in JPE.