

Expectation Shock Simulation with DYNARE: Mark II*

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Abstract

This note demonstrates a revised tool which is designed for conducting an expectation shock simulation easily with DYNARE.

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1 Introduction

Recently, it becomes popular to understand the macroeconomic responses against expectation (news) shock. Beaudry and Portier (2004a, 2004b) analyze a phenomenon in which investment takes place with expectations for higher technology growth in the future, but it turns out not to happen and recession occurs due to stock adjustments. They reproduce such a mechanism working through overinvestment reflecting bubble expectation and its subsequent collapse in the dynamic general equilibrium (henceforth, DGE) model. With this setting, via an expectation on supply side shock, which works like a demand shock and eventually induces overinvestment and its collapse, we can have a realistic cycle of the bubble and its bursting. Therefore, the approaches taken by Beaudry and Portier (2004a, 2004b) have received a lot of attention. According to Rebelo (2005), which summarizes the developments in the RBC modeling to date, it is introduced and praised as one of the most prospective areas in DGE modeling that “Beaudry and Portier (2004a) take an important first step in proposing a model that generates the right comovement in response to news about future increases in productivity. . . . Beaudry and Portier model is an interesting challenge to future research.”

Beaudry and Portier (2004a, 2004b) first show that with the standard RBC model, the comovement in consumption, investment, and labor hours cannot be generated against a news shock on future high technology. This is because expectation about high technology in the future increases the real rate of return as well as creates a wealth effect. Therefore, if the wealth effect surpasses the effect that increases the real rate of return, consumption and leisure increase. However, as labor hours decrease, output level wanes. At the same time, the fact that consumption increases while output decreases reduces investment. On the other hand, if effect on the expected real rate of return is stronger—namely the substitution effect dominates wealth effects—investment and labor hours increase. Since high productivity has not yet been materialized, output growth is smaller than that in investment. Therefore, consumption weakens. Thus, in each case, we cannot have positive comovement in consumption, investment, and labor supply. Beaudry and Portier (2004a, 2004b) claim that complementarity between producing different goods, such as adjustment costs among different sectors, needs to be intensified to have such a comovement.

Yet, multisectoral adjustment cost to intensify complementarity in their paper is the one to express the demand part of resource constraint as the CES aggregator. Therefore, considering the consistency with the SNA scheme, it is not a very realistic adjustment cost. Christiano, Motto and Rostagno (2005) generate positive comovement in consumption, investment, and labor supply

against a news shock for future productivity only with investment intertemporal adjustment cost and habit formation in consumption, which are very realistic and popular mechanisms assumed in the DGE models. Intuitively, they try to increase labor supply by introducing investment adjustment cost and increase in output from raised labor supply should be appropriately divided between consumption and investment. On the other hand, Jaimovich and Rebelo (2006) incorporate variable capital utilization, adjustment costs to investment, and preferences that exhibit a weak short-run wealth effect on the labor supply so that a model can generate an economic expansion following good news about future total factor productivity or investment-specific technical change. Thus, there are growing interest in realistic modelling using expectation shocks, particularly to the future technology.

This note aims at conducting such an expectation shock simulation straightforwardly with Dynare, which is widely used for analyzing DGE models for its very user-friendly operation. A program designed for this purpose, namely `expectation_shock.m` in `expectationshock.zip`, can be applied to any rational expectation model in Dynare form and to any shocks. The structure of this note is as follows. In the next section, following Christiano, Motto and Rostagno (2005), we first construct a model with habit formation in consumption and investment adjustment cost for the simulation of an expectation shock. Then, in Section three, we show how to generate an expectation shock using a canonical form of stochastic process of disturbances. Then, Section four demonstrates the procedure to run such a simulation with `expectation_shock.m`, which is created to generate an expectation shock easily with Dynare. Finally, in Section five, results from an expectation shock simulation are interpreted. A technical back grounds for obtaining required matrices to compute impulse responses against expectation shocks with Dynare are shown in the appendix.

2 Sample Model

A benevolent social planner maximizes instantaneous utility consisting of consumption C , and labor supply h as follows:

$$\frac{[C_t - \exp(b_t) \bar{b} C_{t-1}]^{1 - \exp(\sigma_t) \bar{\sigma}}}{1 - \exp(\sigma_t) \bar{\sigma}} - \frac{\exp(\chi_t) \bar{\chi}}{2} h_t^2,$$

subject to the resource constraint:

$$C_t + I_t \leq [\exp(z_t) h_t]^{1 - \exp(\alpha_t) \bar{\alpha}} K_t^{\exp(\alpha_t) \bar{\alpha}},$$

and the capital formation:

$$K_{t+1} = [1 - \exp(\delta_t)\bar{\delta}] K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t,$$

where b is habit parameter on consumption, σ is the coefficient of the relative risk aversion, χ determines the degree of disutility from labor, I is the investment, z is the labor-augmenting technology and K is the capital stock with α defines the capital share. $S(\cdot)$ is the investment adjustment cost function as:

$$S\left(\frac{I_t}{I_{t-1}}\right) = S'' \left[\frac{1}{2} \exp(\eta_t) \left(\frac{I_t}{I_{t-1}}\right)^2 - \exp(\eta_t) \left(\frac{I_t}{I_{t-1}}\right) + \frac{1}{2} \right].$$

Variables in $\exp(\cdot)$ denote disturbances defined as below. In this note, to be able to conduct simulations with various expectation shocks, we add as many shocks to the model as possible.¹ These shocks are assumed to follow simple stochastic processes:

$$\sigma_t = \rho_\sigma \sigma_{t-1} + \varepsilon_{\sigma,t},$$

$$b_t = \rho_b b_{t-1} + \varepsilon_{b,t},$$

$$\chi_t = \rho_\chi \chi_{t-1} + \varepsilon_{\chi,t},$$

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t},$$

$$\alpha_t = \rho_\alpha \alpha_{t-1} + \varepsilon_{\alpha,t},$$

$$\delta_t = \rho_\delta \delta_{t-1} + \varepsilon_{\delta,t},$$

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_{\eta,t},$$

and subjective discount factor also follows a simple AR process,

$$\beta_t = \rho_\beta \beta_{t-1} + \varepsilon_{\beta,t}.$$

From the first order necessary conditions, we can obtain a model with following 15 equations (1) to (15), where the theoretical stock price q is expressed now as

$$q_t = \frac{\mu_t}{\lambda_t},$$

$$\begin{aligned} & [C_t - \bar{b} \exp(b_t) C_{t-1}]^{-\exp(\sigma_t)\bar{\sigma}} = \lambda_t \quad (1) \\ & + \mathbf{E}_t \exp(\beta_t) \bar{\beta} \bar{b} \exp(b_{t+1}) [C_{t+1} - b \exp(b_{t+1}) C_t]^{-\exp(\sigma_{t+1})\bar{\sigma}}, \end{aligned}$$

¹Therefore, some shocks are of little economic meaning.

$$\exp(\chi_t) \bar{\chi} h_t = \lambda_t [1 - \exp(\alpha_t) \bar{\alpha}] [\exp(z_t) h_t]^{-\exp(\alpha_t) \bar{\alpha}} K_t^{\exp(\alpha_t) \bar{\alpha}}, \quad (2)$$

$$1 = q_t \left\{ \begin{aligned} &1 - S'' \left[\frac{1}{2} \exp(\eta_t) \left(\frac{I_t}{I_{t-1}} \right)^2 - \exp(\eta_t) \left(\frac{I_t}{I_{t-1}} \right) + \frac{1}{2} \right] \\ &- S'' \exp(\eta_t) \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \end{aligned} \right\} \quad (3)$$

$$+ \mathbb{E}_t \exp(\beta_t) \bar{\beta} \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'' \exp(\eta_{t+1}) \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2,$$

$$q_t = \beta_t \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \begin{aligned} &\exp(\alpha_{t+1}) \bar{\alpha} [\exp(z_{t+1}) h_{t+1}]^{1-\exp(\alpha_{t+1}) \bar{\alpha}} K_{t+1}^{\exp(\alpha_{t+1}) \bar{\alpha}-1} \\ &+ q_{t+1} [1 - \exp(\delta_{t+1}) \bar{\delta}] \end{aligned} \right\}, \quad (4)$$

$$C_t + I_t = [\exp(z_t) h_t]^{1-\exp(\alpha_t) \bar{\alpha}} K_t^{\exp(\alpha_t) \bar{\alpha}}, \quad (5)$$

$$K_{t+1} = (1 - \delta_t) K_t + \left\{ 1 - S'' \left[\frac{1}{2} \exp(\eta_t) \left(\frac{I_t}{I_{t-1}} \right)^2 - \exp(\eta_t) \left(\frac{I_t}{I_{t-1}} \right) + \frac{1}{2} \right] \right\} I_t, \quad (6)$$

$$\frac{1}{R_{t+1}} = \beta_t \frac{\mathbb{E}_t \lambda_{t+1}}{\lambda_t} \quad (7)$$

$$\sigma_t = \rho_\sigma \sigma_{t-1} + \varepsilon_{\sigma,t}, \quad (8)$$

$$b_t = \rho_b b_{t-1} + \varepsilon_{b,t}, \quad (9)$$

$$\chi_t = \rho_\chi \chi_{t-1} + \varepsilon_{\chi,t}, \quad (10)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \quad (11)$$

$$\alpha_t = \rho_\alpha \alpha_{t-1} + \varepsilon_{\alpha,t}, \quad (12)$$

$$\delta_t = \rho_\delta \delta_{t-1} + \varepsilon_{\delta,t}, \quad (13)$$

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_{\eta,t}. \quad (14)$$

and

$$\beta_t = \rho_\beta \beta_{t-1} + \varepsilon_{\beta,t}. \quad (15)$$

`pigoucycle.mod.` is the Dynare code for running the above sample model.

3 Expectation Shock

First, we will explain a general solution of the rational expectation model. We then show how to incorporate an expectation shock.²

Generally, a rational expectation model can be represented as:³

$$\begin{aligned} \overline{\alpha}_0 E_t(Z_{t+1} - Z^*) + \overline{\alpha}_1(Z_t - Z^*) + \overline{\alpha}_2(Z_{t-1} - Z^*) \\ + \overline{\beta}_0(S_{t+1} - S^*) + \overline{\beta}_1(S_t - S^*) = 0, \end{aligned} \quad (16)$$

and

$$S_t = S^* + \overline{P}(S_{t-1} - S^*) + \overline{C}\varepsilon_t. \quad (17)$$

where Z is the vector of endogenous variables while S is the vector of shocks. The solution that we would like to obtain is

$$Z_t = Z^* + \overline{A}(Z_{t-1} - Z^*) + \overline{B}(S_t - S^*). \quad (18)$$

By substituting, equations (17) and (18) into (16), we can obtain:

$$\overline{\alpha}_0 \overline{A}^2 + \overline{\alpha}_1 \overline{A} + \overline{\alpha}_2 = 0, \quad (19)$$

and

$$(\overline{\beta}_0 + \overline{\alpha}_0 \overline{B}) \overline{P} + (\overline{\beta}_1 + \overline{\alpha}_1 \overline{B} + \overline{\alpha}_0 \overline{A} \overline{B}) = 0. \quad (20)$$

Matrix \overline{A} and \overline{B} in solutions in equations (17) and (18) are computed by solving the above equations (19) and (20). Especially whether we can obtain unique \overline{A} is dependent on the usual Blanchard and Kahn (1980) condition. \overline{A} and \overline{B} are obtained or computed from the outcome of Dynare simulation in `dr_`.

Simulation with an expectation shock can be materialized by making adjustment to $\overline{\beta}_0$ and $\overline{\beta}_1$ so that we can obtain a new \overline{B} matrix. For simplicity of argument, let us consider a very simple technology shock process z , which is comparable to equations (11) and (17), as follows:

$$z_t = \rho_z z_{t-1} + \xi_{z,t-p} + \varepsilon_{z,t}.$$

With this shock process, we can express a news shock ξ for higher future productivity. As a simple example, here we suppose a situation that we receive a news that “productivity is raised in period 2,” namely $p=2$, today,

²The contents in this subsection are based on Christiano, Motto and Rostagno (2006).

³This is just a difference version of Christiano (2000). Since in our model, there are variables whose steady state value is zero, we use difference rather than log-difference.

but it turns out to be false when period 2 actually comes.⁴ The above equation is represented as follows in canonical form as :

$$\begin{pmatrix} z_t \\ \xi_{z,t} \\ \xi_{z,t-1} \end{pmatrix} = \begin{pmatrix} \rho_z & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_{t-1} \\ \xi_{z,t-1} \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,t} \\ \xi_{z,t} \\ 0 \end{pmatrix} \quad (21)$$

If we add a news shock $\xi_{z,0}$ at period zero,

$$\begin{pmatrix} z_0 \\ \xi_{z,0} \\ \xi_{z,-1} \end{pmatrix} = \begin{pmatrix} \rho_z & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_{-1} \\ \xi_{z,-1} \\ \xi_{z,-2} \end{pmatrix} + \begin{pmatrix} 0 \\ \xi_{z,0} \\ 0 \end{pmatrix}.$$

$\xi_{z,0}$ will not affect z_0 and $E_0 z_1$, but shock on technology at period 2 expected in period zero is now:

$$E_0 \begin{pmatrix} z_2 \\ \xi_{z,2} \\ \xi_{z,1} \end{pmatrix} = \begin{pmatrix} \rho^2 & 1 & \rho \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_0 \\ \xi_{z,0} \\ \xi_{z,-1} \end{pmatrix}.$$

Hence,

$$E_0 z_2 = \xi_{z,0},$$

since

$$z_0 = \xi_{z,-1} = 0.$$

Therefore, the shock on technology at period 2 expected in period zero indeed becomes $\xi_{z,0}$. If such expectation is actually materialized, the simulation is conducted using appropriate S vector and $\overline{\beta}_0^*$, $\overline{\beta}_1^*$ and \overline{B}^* defined as below. On the other hand, once period 2 comes, such a positive shock does not happen actually. $\xi_{z,0}$ is offset by ξ_2 , since $\xi_2 = -\varepsilon_0$. This is depicted as:

$$\begin{pmatrix} z_2 \\ \xi_{z,2} \\ \xi_{z,1} \end{pmatrix} = \begin{pmatrix} \rho_z & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ \xi_{z,1} \\ \xi_{z,0} \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Thus, we can generate such a shock as at period zero and one, technology shock is expected to happen at period 2, but it turns out to be a bubble expectation in period 2.

⁴In simulations below, we also show the case when the initial guess turns out to be true.

This canonical form is exactly equation (17). Therefore, a new shock vector S^* now incorporates an expectation shock terms as $\xi_{z,t}$ as

$$S_t^* = \begin{pmatrix} z_t \\ \xi_{z,t} \\ \xi_{z,t-1} \end{pmatrix}.$$

Therefore, if we appropriately arrange new $\overline{\beta}_0^*$ and $\overline{\beta}_1^*$ from original $\overline{\beta}_0$ and $\overline{\beta}_1$ by adding zero vectors to columns corresponding to $\xi_{z,t}$ in S and rewrite new \overline{P}^* , namely

$$\beta_0^* = \begin{pmatrix} 0 & 0 \\ \beta_0 & 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$\beta_1^* = \begin{pmatrix} 0 & 0 \\ \beta_1 & 0 & 0 \\ 0 & 0 \end{pmatrix},$$

and

$$\overline{P}^* = \begin{pmatrix} \rho_z & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

We can compute new \overline{B}^* matrix from equation (20) using `FindandcheckB.m` in Christiano (2002). We can then obtain impulse responses under an expectation shock with equations (17) and (18).

4 Dynare Procedure

To draw impulse responses against expectation shocks, we need to follow several steps below:

- Download the zip file of Dynare for Matlab from www.ceprenap.cnrs.fr/dynare and install required files in an appropriate folder.
- Decompress `expectationshock.zip` into an appropriate folder.
- Launch Matlab with having a path to the `...Dynare/ Matlab` folder.
- Run the model using Dynare by typing `dynare pigoucycle`; for the case with the sample model in Section two.


```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Definition of Parameters                                           %
% setting (all you need to specify are just 4 parameters below)    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 1. Determine which shock is expectation shock
% 2. Period for expectation is to be realized. Period must be larger than 0
% 3. Length of period to simulate
% 4. Impulse responses in level difference(impulse_index=1) or percentage deviation(impulse_index=2)

expshock='z':
period=4:
simperiod=20:
impulse_index=1:

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

Figure 1: Settings

- Set parameters in `expectation_shock.m`. Below is the setting section in `expectation_shock.m`. You do not have to alter anything other than the area shown in Figure 1 as below. Here, we just need to specify four parameters. The first one is `expshock`. You put the name of the shock which you would like to make an expectation shock. The shock name assigned to be an expectation shock must be consistent with that specified in `dynare mod` file. In this example, standard labor augmenting technology, namely 'z', is assigned to be an expectation shock. The second one is `period`. This specifies the period after when the shock is first expected to be materialized at time zero. The third one is `simperiod` for the whole simulation period. The last one is `impulse_index`. If this is set to 1, program returns the impulse responses as level differences from steady states. On the other hand, if it is 2, those as percentage deviations from steady states are shown. This `impulse_index` is made for the special case when steady states of some endogenous variables are zero so that we can not compute percentage deviations. Therefore, this should usually be set to 2.
- Run `expectation_shock.m`. This will draw two types of impulse responses. The first one is the case when such expectation is actually materialized with `materialized:` in title. The other is the case when such expectation is mirage namely not materialized after all with `not`

materialized: in title.

5 Results

Here, following the arguments in Christiano, Motto and Rostagno (2005), we first check that the standard RBC model without real rigidities cannot generate positive comovement against a news shock. Then, by incorporating investment adjustment cost and habit formation, we demonstrate that business cycles where all show positive comovements can be generated. We herewith draw the impulse responses against an expectation shock to the labor augmenting technology which is expected to happen four quarters later at period zero.⁵ Furthermore, to understand the role of an expectation shock from substitution as well as wealth effects clearly, we show three cases, namely cases with (1) no rigidity, (2) investment adjustment cost, and (3) investment adjustment cost and habit.

Figure 2 demonstrates impulse responses when agents receive a news shock at period zero that “productivity will become higher at period four” when there is neither investment adjustment cost nor habit formation using `expectation_shock.m`. The blue line shows when the expectation is actually materialized while the red line is the case when it is not. As explained in the introduction, increased discounted present value of the real wage induces higher wealth effect. Under the calibration in this paper, since the intertemporal elasticity of substitution is not huge, the wealth effect dominates the substitution effect. Hence, consumption and leisure are increased. Increase in leisure naturally results in lower labor supply. Furthermore, since higher productivity is not yet materialized initially, investment decreases.⁶ Without investment adjustment cost and habit formation in consumption, we cannot have positive comovement in consumption, investment, and labor supply against a news shock for higher productivity in the future.

Figure 3 demonstrates impulse responses when investment adjustment cost is incorporated. Figure 2 shows that it is desirable to increase investment massively right after confirming the increase in technology. On the other hand, if investment adjustment cost is considered, as in Figure 3, level of investment is gradually raised once agents receive a news about higher productivity in the future to avoid such a sudden increase in investment observed in Figure 2. Thus, investment as well as labor supply increase against an expectation shock. However, the growth rate of productivity is lower than

⁵For calibration of parameters, see `pigoucycle.mod`.

⁶A decrease in investment can also be explained by the elasticity of substitution between labor and capital.

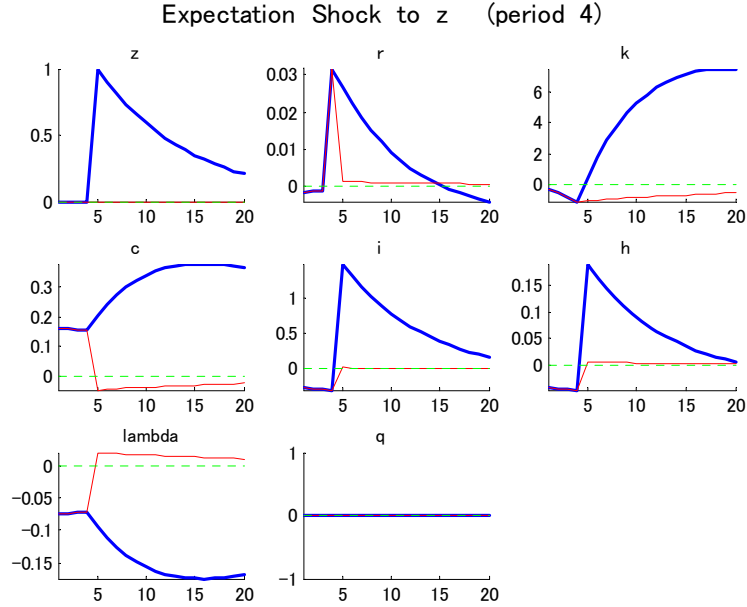


Figure 2:

of investment until period eight. If higher productivity does not materialize, consumption is lowered since the resource constraint needs to be satisfied.

Figure 4 shows when habit formation on consumption is further incorporated. Consumption plummets when agents actually perceive higher productivity in Figure 4. With habit formation, however, since households dislike ups and downs in consumption, consumption exhibits a smooth, increasing trend.

In the DGE model, responses against such shocks as monetary policy shock and technology shock are considered to be optimal responses to shocks. Therefore, it is almost impossible to reproduce such a phenomenon as overinvestment. For example, a situation when expansionary monetary policy induces investment and interest rate is raised afterward should not be interpreted as overinvestment, but as mistakes in monetary policy implementation. However, as shown in above figures, expectation about future high technology that has not yet materialized can intensify capital formation, and this expectation will turn out to be wrong. The initial increase in investment is indeed considered to be overinvestment. We can reproduce a familiar story that “investment is facilitated by the optimistic expectation about the future, but it does not materialize. As a result, initial increase in investment

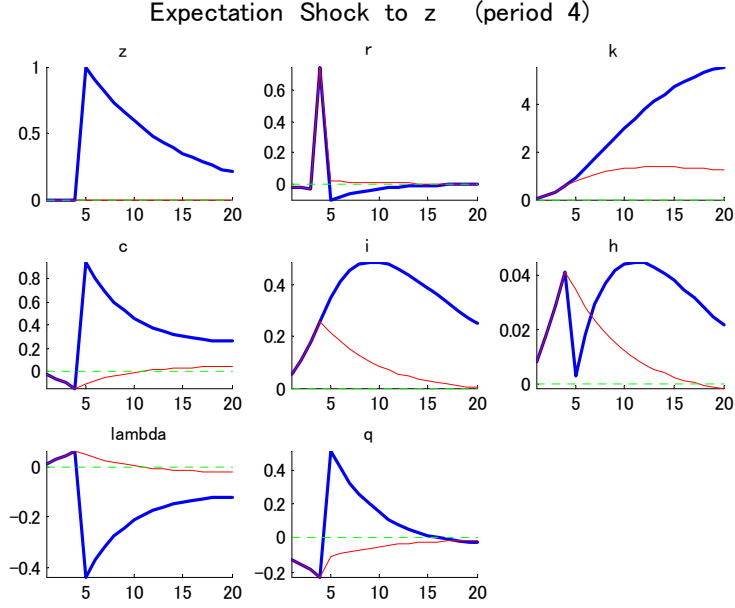


Figure 3: With investment adjustment cost

is indeed interpreted as overinvestment.”

Yet, in Figure 4, although we can monitor positive comovements in consumption, investment and labor supply against a positive news shock about future technology, theoretical stock price decreases. This may seem counter-intuitive but becomes clearer by looking into equations (3) and (4) carefully:

$$q_t = \sum_{i=1}^{\infty} E_t \left(\frac{1}{\prod_{j=1}^i R_{t+j}} \right) (1 - \delta)^{i-1} \alpha [\exp(z_{t+i}) h_{t+i}]^{1-\alpha} K_{t+i}^{\alpha}, \quad (22)$$

$$q_t = \frac{1}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}} - \frac{q_{t+1}}{R_{t+1}} \frac{S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}}. \quad (23)$$

Although we solve the optimization problem of the social planner, above equations can be interpreted in line with the competitive equilibrium with the representative household and capital producer. Equation (22) transformed from equation (4) is considered to be the capital demand function, where households equates the theoretical price of capital to the present discounted value of future dividends. On the other hand, equation (23) from equation

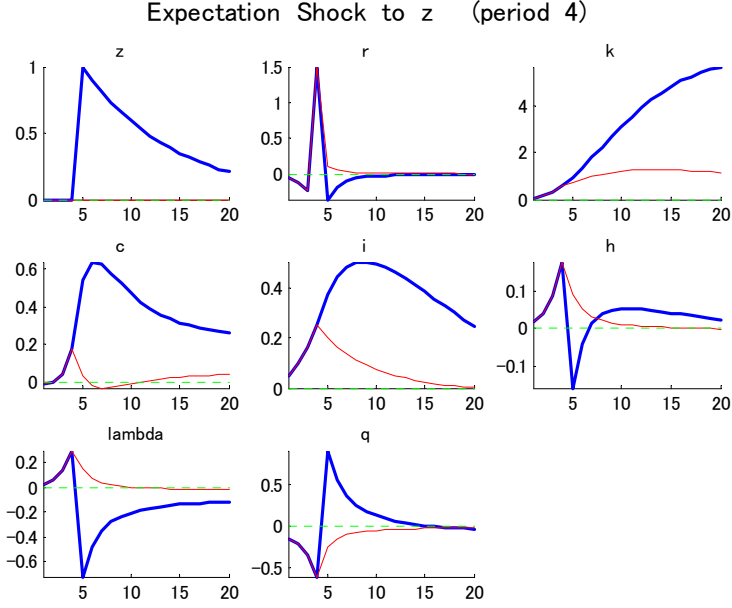


Figure 4: With investment adjustment cost and habit

(3) is the capital supply function. Capital producers set the price of capital, namely the left hand side, based on the marginal cost of producing a unit of capital, namely the right hand side.

Figure 5, which draws the relationship between equations (22) and (23), clarifies the dynamic transition of theoretical capital price from period zero to one. Economy is at q_0 initially. After receiving a news shock for higher future technology, the capital demand curve shifts upwards due to the expected increase in dividend, namely (i). Such upward shift of the demand curve is, however, mitigated by an increase in the real interest rate determined from the intertemporal ratio of the marginal utility from consumption, as shown by (ii). Concerning the supply side developments on the other hand, increased investment reacting to a positive news shock naturally raises marginal cost contemporaneously as (iii) and is understood from the first term in the right hand side of equation (23). Yet, an increase in investment today reduces the adjustment cost supposed to be incurred due to higher investment growth in the future. This is represented as the second term in equation (23). Therefore, as shown in (iv), the capital supply curve shifts downwards even investment in period one is increased. In aggregate, reflecting such demand and supply conditions, theoretical stock price is lowered from q_0 to

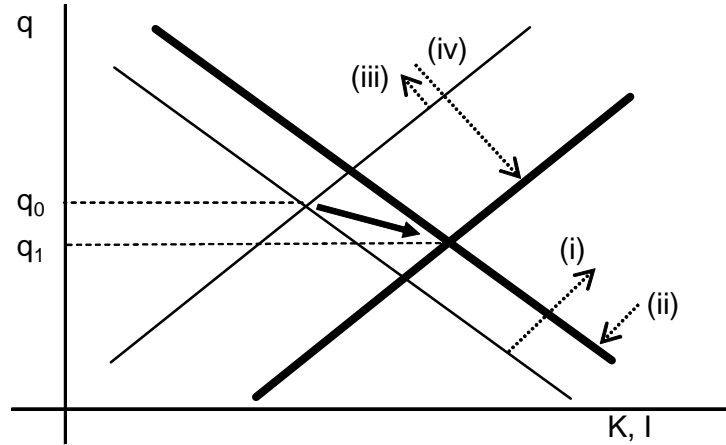


Figure 5: Capital Equilibrium

q_1 in response to an expectation shock for future higher productivity. To have a stock price bubble in this setting, Christiano, Motto and Rostagno (2005) incorporate sticky prices and the Taylor type instrument rule. A news shock about high future productivity implies the lower marginal cost in the future. If price setting is mostly forward looking, namely with less indexation and barrier to acquiring new information, this lowers current inflation rates. Hence, nominal as well as real interest rates are lowered according to the Taylor type instrument rule reacting aggressively to inflation developments. This shifts the capital demand curve in Figure 8 outwards. As a result, stock price boom can happen after an expectation shock hits the economy in their setting.

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Appendix: Matrices in Dynare and expectation_shock.m

In Dynare, from `jacobia_` in `dr1.m`, we can arrange the second order Matrix difference equation as follows:

$$\begin{aligned} \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} E_t \begin{pmatrix} Z_{t+1} - Z^* \\ S_{t+1} - S^* \end{pmatrix} + \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} \begin{pmatrix} Z_t - Z^* \\ S_t - S^* \end{pmatrix} \\ + \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \begin{pmatrix} Z_{t-1} - Z^* \\ S_{t-1} - S^* \end{pmatrix} + \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} \varepsilon_t = 0. \end{aligned} \quad (24)$$

At the same time, `dr_.ghx` and `dr_.ghu` obtained from the Dynare output, we can express the backward state space form of the rational expectation model:

$$\begin{pmatrix} Z_t \\ S_t \end{pmatrix} = \begin{pmatrix} Z^* \\ S^* \end{pmatrix} + \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} Z_{t-1} - Z^* \\ S_{t-1} - S^* \end{pmatrix} + \begin{pmatrix} J_1 \\ J_2 \end{pmatrix} \varepsilon_t. \quad (25)$$

Since any forward looking model can be expressed in equations (16) and (17), we can obtain identities as follows:

$$\begin{aligned} D_{11} &= \overline{\alpha_0}, \\ E_{11} &= \overline{\alpha_1}, \\ F_{11} &= \overline{\alpha_2}, \\ D_{12} &= \overline{\beta_0}, \\ E_{12} &= \overline{\beta_1}, \\ E_{22}^{-1} F_{22} &= \overline{P}, \\ G_2 &= \overline{C}, \\ D_{21} &= E_{21} = F_{21} = 0, \\ D_{22} &= 0, \end{aligned}$$

and

$$G_1 = 0.$$

On the other hand, from equations (17), (18) and (25), below identities must also be satisfied:

$$\begin{aligned} H_{11} &= \overline{A}, \\ H_{12} P^{-1} &= \overline{B}, \end{aligned}$$

$$H_{22} = \bar{P},$$

$$J_2 = \bar{C},$$

$$H_{21} = 0,$$

and

$$J_1 - H_{12}P^{-1}C = 0.$$

Furthermore, equations (19) and (20) mean that below must hold as well:

$$D_{11}H_{11}^2 + E_{11}H_{11} + F_{11} = 0,$$

and

$$(D_{12} + D_{11}H_{12}H_{22}^{-1})H_{22} + E_{12} + E_{11}H_{12}H_{22}^{-1} + D_{11}H_{11}H_{12}H_{22}^{-1} = 0.$$

Thus, we can obtain \bar{A} , \bar{B} , \bar{C} and \bar{P} in equations (17) and (18) required for conducting an expectation shock simulation. At the same time, whether above identities hold or not are also checked in `expectation_shock.m`.