

CDFLIB90
Fortran 95 Routines for Cumulative Distribution Functions,
Their Inverses, and More.
User's Guide

Barry W. Brown
James Lovato
Kathy Russell

Conversion to Fortran 95
Dan Serachitopol

This work supported by grant CA11672 from the National Cancer Institute. Copyright 2002 to:

The University of Texas M. D. Anderson Cancer Center
Department of Biomathematics, Box 237
1515 Holcombe Boulevard
Houston, TX 77030

Contact: Barry W. Brown, bwb@mdanderson.org

1 Technicalities

1.1 Obtaining the Code

The source for this code (and all code written by this group) can be obtained from the following URL:

<http://odin.mdacc.tmc.edu/anonftp/>

1.2 Legalities

We place our efforts in writing this package in the public domain. However, code from ACM publications is subject to the ACM policy (below).

1.3 References

1.3.1 Incomplete Beta

DiDinato, A. R. and Morris, A. H. (1993) “Algorithm 708: Significant Digit Computation of the Incomplete Beta Function Ratios.” *ACM Trans. Math. Softw.* 18, 360-373.

1.3.2 Incomplete Gamma

DiDinato, A. R. and Morris, A. H. (1986) “Computation of the incomplete gamma function ratios and their inverse.” *ACM Trans. Math. Softw.* 12, 377-393.

1.3.3 Cumulative Normal

Cody, W.D. (1993). “ALGORITHM 715: SPECFUN - A Portable FORTRAN Package of Special Function Routines and Test Drivers” *ACM Trans. Math. Softw.* 19, 22-32.

1.3.4 Inverse Normal

“Algorithm AS241” (1988) *Appl. Statist.* 37, NO. 3, 477-484.

1.3.5 Finding a Zero of a Monotone Function

Alefeld, G. E., Potra, F. A., Shi, Y. (1995) “Algorithm 748: Enclosing Zeros of Continuous Functions.”, by G. E. Alefeld, F. A. Potra, YiXun Shi, *ACM Trans. Math. Softw.*, 21, No. 3, 327-344

1.4 ACM Policy on Use of Code

Here is the software Policy of the ACM.

Submittal of an algorithm for publication in one of the ACM Transactions implies that unrestricted use of the algorithm within a computer is permissible. General permission to copy and distribute the algorithm without fee is granted provided that the copies are not made or distributed for direct commercial advantage. The ACM copyright notice and the title of the publication and its date appear, and notice is given that copying is by permission of the Association for Computing Machinery. To copy otherwise, or to republish, requires a fee and/or specific permission.

Krogh, F. (1997) "Algorithms Policy." *ACM Tran. Math. Softw.* 13, 183-186.

We do not know the policy of the Royal Statistical Society; they have discontinued publishing algorithms. However, they made a number of these programs available on Statlib on condition that there be no charge for their distribution.

Here is our standard disclaimer.

NO WARRANTY

WE PROVIDE ABSOLUTELY NO WARRANTY OF ANY KIND EITHER EXPRESSED OR IMPLIED, INCLUDING BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE. THE ENTIRE RISK AS TO THE QUALITY AND PERFORMANCE OF THE PROGRAM IS WITH YOU. SHOULD THIS PROGRAM PROVE DEFECTIVE, YOU ASSUME THE COST OF ALL NECESSARY SERVICING, REPAIR OR CORRECTION.

IN NO EVENT SHALL THE UNIVERSITY OF TEXAS OR ANY OF ITS COMPONENT INSTITUTIONS INCLUDING M. D. ANDERSON HOSPITAL BE LIABLE TO YOU FOR DAMAGES, INCLUDING ANY LOST PROFITS, LOST MONIES, OR OTHER SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF THE USE OR INABILITY TO USE (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA OR ITS ANALYSIS BEING RENDERED INACCURATE OR LOSSES SUSTAINED BY THIRD PARTIES) THE PROGRAM.

(Above NO WARRANTY modified from the GNU NO WARRANTY statement.)

2 Introduction to CDFLIB90

2.1 Modules

CDFLIB90 contains routines that calculate the cumulative distribution function (CDF), the inverse CDF, and the values of one parameter of the distribution given the value of the CDF and of the other parameters.

REAL TYPE. The only type of Fortran REAL used in this library is DOUBLE PRECISION, which we term **dpkind**.

CDFLIB90 consists of Fortran 95 modules, named as follows:

cdf_<distribution>_mod

Each program unit using one or more of these module must have the appropriate USE statement(s).

The following table shows the possible values of <distribution> and the corresponding statistical distribution.

<distribution>	Statistical Distribution
beta	beta
binomial	binomial
chisq	chi-squared
f	f
gamma	gamma
nc_chisq	noncentral chi-squared
nc_f	noncentral f
nc_t	noncentral t
neg_binomial	negative binomial
normal	normal
poisson	poisson
t	t

2.2 Routines

Each module contains four user accessible (PUBLIC) routines:

SUBROUTINE CDF_<distribution>(WHICH, CUM, CCUM, X, <PARAMS>, STATUS, CHECK_INPUT.) Calculates the value of any one of its parameters from the values of the others.

REAL (dpkind) FUNCTION CUM_<distribution>(X, <PARAMS>, STATUS, CHECK_INPUT.) Calculates argument CUM.

REAL (dpkind) FUNCTION CCUM_<distribution>(X, <PARAMS>, STATUS, CHECK_INPUT) Calculates argument CCUM.

REAL (dpkind) FUNCTION INV_<distribution>(CUM, CCUM, <PARAMS>, STATUS, CHECK_INPUT)

2.3 Arguments

INTEGER, INTENT(IN):: WHICH. Determines which parameter is to be calculated from the values of the other.

REAL (dpkind) :: CUM, CCUM. The CDF and complement of the CDF, 1-CUM.

REAL (dpkind) :: X. The name of this argument varies with the distribution.

The upper limit of integration or summation for the CDF. The lower limit is the lower limit of the support of the distribution.

Note that X is always DOUBLE PRECISION, even when it appears that it should take only integer values, as in the binomial distribution. This is because discrete distribution is embedded in continuous distributions. The binomial and negative binomial are calculated as a special case of the beta distribution; the Poisson distribution is calculated as a special case of the gamma distribution.

One consequence: when X is computed, it need not and usually does not take on an integer value for discrete distributions.

REAL (dpkind) :: <PARAMS>. The parameters of the distribution, e.g., the mean and variance of the normal distribution or the degrees of freedom of the t distribution.

INTEGER, INTENT(OUT), OPTIONAL :: STATUS. The status of the calculation. **Although this argument is optional, it is strongly recommended that all code include it and its value checked by the calling program.**

Zero indicates success.

A small negative number indicates that the argument whose order is the absolute value of status is out of bounds.

A small positive value generally indicates that input arguments that should add (approximately) to one don't.

The value '10' indicates that the cumulative function is obtained from another distribution (e.g., the binomial is calculated as an incomplete beta) and there was an error in the other distribution. This really should not occur.

Should the status be any of the values above and the argument STATUS is not present, then the program aborts (STOP) with an error message.

A value of 50 indicates that the answer, should it exist exceeds the upper search bound of the routine; a value of -50 indicates that the lower search bound is exceeded. In either case, the upper or lower search bound respectively is returned as the answer. The program does not abort in this case should STATUS not be present in the calling sequence.

Out of bound errors are the ones most commonly encountered because for the discrete distributions (binomial, negative binomial, and Poisson), the cdf at 0 is positive not zero. Hence the inverse cdf does not exist for sufficiently small values of the cdf.

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If this argument is present and has the value .FALSE. then input values are not checked for legality. Generally, this is a bad idea and can lead to difficult to find errors. The argument is included for those cases in which arguments are checked at a higher level.

INPUT AND SEARCH RANGES. Each input argument to one of the CDF90 routines must be in the *range* of values. Values outside the range will be signalled as an error in the STATUS variable or cause an abort if STATUS is not present. **NOTE:** We show the range as input range for the argument WHICH because WHICH is always input and never calculated.

For most distributions (exceptions, the normal and gamma), finding a parameter value producing a specified cdf value is done by a search within the range of allowable values. If an answer is not found within this range, an error is signalled in the STATUS variable, or the program aborts if STATUS is not present in the calling sequence.

Generally, the allowable range of the CUM and CCUM arguments is: $10^{10} : 1 - 10^{10}$. Exceptions occur when one or both sides of the support of the distribution is finite. Then the search range includes 0 or 1 or both (depending on which end(s) of the support is finite).

We somewhat arbitrarily limit the range of values of the various degrees of freedom arguments to

Range: $[10^{-3} : 10^{10}]$

A degrees of freedom argument of zero frequently leads to an undefined distribution. The noncentrality parameters of the noncentral distributions are limited to

Range: $[0, 10^4 : T]$

the upper limit is imposed because the cdfs of these distributions are calculated as infinite series in the noncentrality parameter. The number of terms to be evaluated becomes large with this parameter.

3 cdf_beta_mod

SUBROUTINE CDF_BETA(WHICH, CUM, CCUM, X, CX, A, B, STATUS,
CHECK_INPUT)

REAL (dpkind) FUNCTION CUM_BETA(X, A, B, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CCUM_BETA(X, A, B, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION INV_BETA(CUM, CCUM, A, B, STATUS, CHECK_INPUT)

3.1 The Distribution

The density of the beta distribution is defined on x in $[0, 1]$ and is proportional to

$$x^a(1-x)^b$$

3.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next four arguments is to be calculated.

Input Range: [1 : 4]

1. CUM and CCUM
2. X and CX
3. A
4. B

REAL (dpkind), OPTIONAL :: CUM. The CDF of the beta distribution.

Range: [0 : 1]

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the beta distribution.

Range: [0 : 1]

REAL (dpkind), OPTIONAL :: X. The upper limit of integration of the beta density. The lower limit is 0.

Range: [0 : 1]

REAL (dpkind), OPTIONAL :: CX. One minus the upper limit of integration of the beta density. The lower limit is 0.
Range: [0 : 1]

REAL (dpkind) :: A. The first parameter of the beta density.
Range: [10^{-10} : 10^{10}]

REAL (dpkind) :: B. The second parameter of the beta density.
Range: [10^{-10} : 10^{10}]

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 X outside range

-5 CX outside range

-6 A outside range

-7 B outside range

3 CUM + CCUM is not nearly one

4 X + CX is not nearly one

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

NOTE: CUM and CCUM and also X and CX must add to (nearly) one.

4 `cdf_binomial_mod`

SUBROUTINE CDF_BINOMIAL(WHICH, CUM, CCUM, S, N, PR, CPR, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CUM_BINOMIAL(S, N, PR, CPR, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CCUM_BINOMIAL(S, N, PR, CPR, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION INV_BINOMIAL(CUM, CCUM, N, PR, CPR, STATUS, CHECK_INPUT)

4.1 The Distribution

The density of the binomial distribution provides the probability of S successes in N independent trials, each with probability of success PR .

The density is proportional to

$$PR^S(1 - PR)^{N-S}$$

The binomial is extended to non-integer values via the connection between the cumulative binomial and the incomplete beta.

4.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next four arguments is to be calculated.

Input Range: [1 : 4]

1 CUM and CCUM

2 S

3 N

4 PR and CPR

REAL (dpkind), OPTIONAL :: CUM. The CDF of the binomial distribution, i.e., the probability of 0 to S successes in N trials.

Range: [0 : 1 - 10⁻¹⁰]

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the binomial distribution.

Range: [10⁻¹⁰ : 1]

REAL (dpkind) :: S. The upper limit of summation of the binomial density. Note that S must be less than or equal to N.
Range: [0 : 10¹⁰]

REAL (dpkind) :: N. The number of independent trials generating the binomial density. N must be greater than or equal to S.
Range: [0 : 10¹⁰]

REAL (dpkind), OPTIONAL :: PR. The probability of success in each independent trial.
Range: [0 : 1]

REAL (dpkind), OPTIONAL :: CPR. One minus the probability of success in each independent trial; the probability of failure in each trial.
Range: [0 : 1]

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 S outside range

-5 N outside range

-6 PR outside range

-7 CPR outside range

3 CUM + CCUM is not nearly one

4 PR + CPR is not nearly one

5 S not between 0 and N.

10 The cumulative binomial is calculated as an incomplete beta. This error indicates an error in the incomplete beta. This should not happen.

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

NOTE: CUM and CCUM and also PR and CPR must add to (nearly) one.

5 cdf_chisq_mod

```
SUBROUTINE CDF_CHISQ( WHICH, CUM, CCUM, X, DF, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION CUM_CHISQ(X, DF, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION CCUM_CHISQ(X, DF, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION INV_CHISQ( CUM, CCUM, DF, STATUS, CHECK_INPUT)
```

5.1 The Distribution

The chi-squared distribution is the distribution of the sum of squares of DF independent unit (mean=0, sd=1) normal deviates.

The density is defined on x in $[0, \infty)$ and is proportional to

$$x^{(DF-2)/2} \exp(-x/2)$$

5.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next three arguments is to be calculated.

Input Range: [1 : 3]

1. CUM and CCUM
2. X
3. DF

REAL (dpkind), OPTIONAL :: CUM. The CDF of the chi-squared distribution.

Range: $[0 : 1 - 10^{-10}]$

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the chi-squared distribution.

Range: $[10^{-10} : 1]$

REAL (dpkind) :: X. The upper limit of integration of the chi-squared density. The lower limit is 0.

Range: $[0 : 10^{100}]$

REAL (dpkind) :: DF. The degrees of freedom of the chi-squared distribution.
Range: $[10^{-3} : 10^{10}]$

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 X outside range

-5 DF outside range

3 CUM + CCUM is not nearly one

4 X + CX is not nearly one

10 The cumulative chi-squared is computed as an incomplete beta distribution. This value indicates an error in the incomplete beta code. It really shouldn't happen.

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

NOTE: CUM and CCUM and also X and CX must add to (nearly) one.

6 cdf_f_mod

```
SUBROUTINE CDF_F( WHICH, CUM, CCUM, F, DFN, DFD, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION CUM_F(F, DFN, DFD, STATUS, CHECK_INPUT)
REAL (dpkind)FUNCTION CCUM_F(F, DFN, DFD, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION INV_F( CUM, CCUM, DFN, DFD, STATUS, CHECK_INPUT)
```

6.1 The Distribution

F is the distribution of the ratio of two independent random variables. The numerator random variable is distributed as chi-squared with DF degrees of freedom divided by DF. The denominator random variable is distributed as chi-squared with DFD degrees of freedom divided by DFD.

The density of the f distribution is defined on x in $[0, \infty]$ and is proportional to

$$\frac{x^{(DFN-2)/2}}{[1 + (DFN/DFD)x]^{(DFN+DFD)/2}}$$

6.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next four arguments is to be calculated.

Input Range: [1 : 2]

1. CUM and CCUM
2. F

NOTE: DFN and DFD will not be computed because CUM is not monotone in either argument.

REAL (dpkind), OPTIONAL :: CUM. The CDF of the f distribution.

Range: [0 : 1 - 10⁻¹⁰]

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the f distribution.

Range: [10⁻¹⁰ : 1]

REAL (dpkind) :: F. The upper limit of integration of the f density. The lower limit is 0.

Input Range: $[0 : 10^{100}]$

REAL (dpkind) :: DFN. The numerator degrees of freedom.

Range: $[10^{-3} : 10^{10}]$

REAL (dpkind) :: DFD. The denominator degrees of freedom.

Range: $[10^{-3} : 10^{10}]$

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 F outside range

-5 DFN outside range

-6 DFD outside range

3 CUM + CCUM is not nearly one

10 The cumulative F is computed as an incomplete beta distribution. This value indicates an error in the incomplete beta code. It really shouldn't happen.

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

NOTE: CUM and CCUM must add to (nearly) one.

NOTE: The value of the CDF of the f distribution is not necessarily monotone in either degree of freedom argument. There may thus be two values that provide a given DCF value. This routine assumes monotonicity and will find an arbitrary one of the two values.

7 `cdf_gamma_mod`

SUBROUTINE CDF_GAMMA(WHICH, CUM, CCUM, X, SHAPE, SCALE,
STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CUM_GAMMA(X, SHAPE, SCALE, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CCUM_GAMMA(X, SHAPE, SCALE, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION INV_GAMMA(CUM, CCUM, SHAPE, SCALE,
STATUS, CHECK_INPUT)

7.1 The Distribution

The density of the GAMMA distribution is proportional to:

$$(x/SCALE)^{SHAPE-1} \exp(-x/SCALE)$$

7.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next four arguments is to be calculated.

Input Range: [1 : 4]

1. CUM and CCUM
2. X
3. SHAPE
4. SCALE

REAL (dpkind), OPTIONAL :: CUM. The CDF of the gamma distribution.
Range: [0 : 1 - 10⁻¹⁰]

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the gamma distribution.
Range: [10⁻¹⁰ : 1]

REAL (dpkind) :: X. The upper limit of integration of the gamma density.
The lower limit is 0.
Range: [0 : 10¹⁰⁰]

REAL (dpkind) :: SHAPE. The shape parameter of the distribution.
Range: $[10^{-10} : 10^{100}]$

REAL (dpkind) :: SCALE. The scale parameter of the distribution.
Range: $[10^{-10} : 10^{100}]$

INTEGER, INTENT(OUT) :: STATUS. Return code.

- 1 WHICH outside input range
- 2 CUM outside range
- 3 CCUM outside range
- 4 X outside range
- 5 SHAPE outside range
- 6 SCALE outside range
- 3 CUM + CCUM is not nearly one
- 5 Some error in inverse gamma routine

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

NOTE: CUM and CCUM must add to (nearly) one.

8 `cdf_neg_binomial_mod`

SUBROUTINE CDF_NEG_BINOMIAL(WHICH, CUM, CCUM, F, S, PR, CPR,
STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CUM_NEG_BINOMIAL(F, S, N, PR, CPR, STA-
TUS, CHECK_INPUT)

REAL (dpkind)FUNCTION CCUM_NEG_BINOMIAL(F, S, N, PR, CPR, STA-
TUS, CHECK_INPUT)

REAL (dpkind) FUNCTION INV_NEG_BINOMIAL(CUM, CCUM, N, PR, CPR,
STATUS, CHECK_INPUT)

8.1 The Distribution

The density of the negative binomial distribution provides the probability of precisely F failures before the S 'th success in independent binomial trials, each with probability of success PR .

The density is

$$\binom{F + S - 1}{S - 1} PR^S (1 - PR)^F$$

The cumulative distribution function is the probability of F or fewer failures before the F 'th success.

The negative binomial is extended to non-integer values of F via the relation between the cumulative distribution function of the negative binomial and the incomplete beta function.

8.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next four arguments is to be calculated.

Input Range: [1 : 4]

1 CUM and CCUM

2 F

3 S

4 PR and CPR

REAL (dpkind), OPTIONAL :: CUM. The CDF of the negative-binomial distribution.

Range: $[0 : 1 - 10^{-10}]$

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the binomial distribution.

Range: $[10^{-10} : 1]$

REAL (dpkind) :: F. The number of failures before the S'th success.

Range: $[0 : 10^{10}]$

REAL (dpkind) :: S. The number of successes to occur.

Input Range: $[0 : 10^{10}]$

REAL (dpkind) :: PR. The probability of success in each independent trial.

Range: $[0 : 1]$

REAL (dpkind) :: CPR. One minus the probability of success in each independent trial; the probability of failure in each trial.

Range: $[0 : 1]$

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 F outside range

-5 S outside range

-6 PR outside range

-7 CPR outside range

3 CUM + CCUM is not nearly one

4 PR + CPR is not nearly one

10 The cumulative negative binomial is computed as an incomplete beta.
This value of STATUS indicates an error in the incomplete beta routine.
It really shouldn't happen.

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

NOTE: CUM and CCUM and also PR and CPR must add to (nearly) one.

9 cdf_nc_chisq_mod

SUBROUTINE CDF_NC_CHISQ(WHICH, CUM, CCUM, X, DF, PNONC, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CUM_NC_CHISQ(X, DF, PNONC, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CCUM_NC_CHISQ(X, DF, PNONC, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION INV_NC_CHISQ(CUM, CCUM, DF, PNONC, STATUS, CHECK_INPUT)

9.1 The Distribution

The noncentral chi-squared distribution is the sum of DF independent normal distributions with unit standard deviations, but possibly non-zero means . Let the mean of the i th normal be δ_i . Then $PNONC = \sum_i \delta_i$.

9.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next three arguments is to be calculated.

1. CUM and CCUM
2. DF
3. PNONC

REAL (dpkind), OPTIONAL :: CUM. The CDF of the noncentral chi-squared distribution.
Range: $[0 : 1 - 10^{-10}]$

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the non-central chi-squared distribution.
Range: $[10^{-10} : 1]$

REAL (dpkind) :: X. The upper limit of integration of the noncentral chi-squared distribution. (The lower limit is 0.) Range: $[0 : 10^{100}]$

REAL (dpkind) :: DF. The degrees of freedom of the noncentral chi-squared distribution.

Input Range: $[10^{-3} : 10^{10}]$

REAL (dpkind) :: PNONC. The noncentrality parameter.

Range: $[0 : 10^4]$

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 X outside range

-5 DF outside range

-6 PNONC outside range

3 CUM + CCUM is not nearly one

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

NOTE: CUM and CCUM must add to (nearly) one.

10 cdf_nc_f_mod

SUBROUTINE CDF_F(WHICH, CUM, CCUM, F, DFN, DFD, PNONC, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CUM_F(F, DFN, DFD, PNONC, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CCUM_F(F, DFN, DFD, PNONC, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION INV_F(CUM, CCUM, DFN, DFD, PNONC, STATUS, CHECK_INPUT)

10.1 The Distribution

The noncentral F is the distribution of the ratio of two independent random variables. The numerator random variable is distributed as a noncentral chi-squared with DFN degrees of freedom and noncentrality parameter PNONC divided by DFN. The denominator random variable is distributed as a (central) chi-squared with DFD degrees of freedom divided by DFD.

10.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next five arguments is to be calculated.

Input Range: [1 : 2]

1. CUM and CCUM
2. F

NOTE: DFN and DFD will not be computed because CUM is not monotone in either argument.

REAL (dpkind), OPTIONAL :: CUM. The CDF of the noncentral f distribution.

Range: [0 : $1 - 10^{-10}$]

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the non-central f distribution.

Range: [10^{-10} : 1]

REAL (dpkind) :: F. The upper limit of integration of the noncentral f density.
The lower limit is 0.
Range: [0 : 10¹⁰⁰]

REAL (dpkind) :: DFN. The numerator degrees of freedom.
Range: [10⁻³ : 10¹⁰]

REAL (dpkind) :: DFD. The denominator degrees of freedom.
Range: [10⁻³ : 10¹⁰]

REAL (dpkind) :: PNONC. The noncentrality parameter.
Range: [0 : 10⁴]

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 F outside range

-5 DFN outside range

-6 DFD outside range

3 CUM + CCUM is not nearly one

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

NOTE: CUM and CCUM must add to (nearly) one.

11 cdf_nc_t_mod

```
SUBROUTINE CDF_T( WHICH, CUM, CCUM, T, DF, PNONC, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION CUM_T(T, DF, PNONC, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION CCUM_T(T, DF, PNONC, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION INV_T( CUM, CCUM, DF, PNONC, STATUS, CHECK_INPUT)
```

11.1 The Distribution

The noncentral T is the distribution of the ratio of two independent random variables. The numerator random variable is distributed as a normal distribution with mean PNONC and variance 1. The denominator random variable is distributed as a the square root of a (central) chi-squared with DF degrees of freedom divided by DF.

11.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next arguments is to be calculated.

Input Range: [1 : 4]

1. CUM and CCUM
2. T
3. DF
4. PNONC

REAL (dpkind), OPTIONAL :: CUM. The CDF of the noncentral t distribution.

Range: [10^{-10} : $1 - 10^{-10}$]

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the non-central t distribution.

Range: [10^{-10} : $1 - 10^{-10}$]

REAL (dpkind) :: T. The upper limit of integration of the noncentral t density. The lower limit is $-\infty$.

Range: [-10^{100} : 10^{100}]

REAL (dpkind) :: DF. The degrees of freedom.
Range: $[10^{-3} : 10^{10}]$

REAL (dpkind) :: PNONC. The noncentrality parameter.
Range: $[0 : 10^4]$

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 T outside range

-5 DF outside range

-6 PNONC outside range

3 CUM + CCUM is not nearly one

10 The noncentral t calculation uses the central t calculation which is reduced to the calculation of an incomplete beta. This value of STATUS indicates an error in the incomplete beta calculation. It really shouldn't happen.

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

12 cdf_normal_mod

SUBROUTINE CDF_NORMAL(WHICH, CUM, CCUM, X, MEAN, SD, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CUM_NORMAL(X, MEAN, SD, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CCUM_NORMAL(X, MEAN, SD, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION INV_NORMAL(CUM, CCUM, MEAN, SD, STATUS, CHECK_INPUT)

12.1 The Distribution

The density of the normal distribution is proportional to

$$\exp\left(-\frac{(X - MEAN)^2}{2SD^2}\right)$$

12.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next arguments is to be calculated.

Input Range: [1 : 4]

1. CUM and CCUM
2. X
3. MEAN
4. SD

REAL (dpkind), OPTIONAL :: CUM. The CDF of the normal distribution.

Range: $[10^{-10} : 1 - 10^{-10}]$

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the normal distribution.

Range: $[10^{-10} : 1 - 10^{-10}]$

REAL (dpkind) :: X. The upper limit of integration of the normal density.

The lower limit is $-\infty$.

Range: $[-10^{100} : 10^{100}]$

REAL (dpkind), OPTIONAL:: MEAN. The mean of the normal distribution. If omitted, the value 0 is used.
Range: $[-10^{100} : 10^{100}]$

REAL (dpkind), OPTIONAL:: SD. The standard deviation of the normal distribution. If omitted, the value 1 is used.
Range: $[10^{-10} : 10^{100}]$

INTEGER, INTENT(OUT), OPTIONAL :: STATUS. Return code.

- 1 WHICH outside input range
- 2 CUM outside range
- 3 CCUM outside range
- 4 X outside range
- 5 MEAN outside range
- 6 SD outside range
- 3 CUM + CCUM is not nearly one

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

13 cdf_poisson_mod

SUBROUTINE CDF_POISSON(WHICH, CUM, CCUM, S, LAMBDA, STATUS,
CHECK_INPUT)

REAL (dpkind) FUNCTION CUM_POISSON(S, LAMBDA, STATUS, CHECK_INPUT)

REAL (dpkind) FUNCTION CCUM_POISSON(S, LAMBDA, STATUS, CHECK_INPUT)

FUNCTION INV_POISSON(CUM, CCUM, LAMBDA, STATUS, CHECK_INPUT)

13.1 The Distribution

The density of the Poisson distribution (probability of observing S events) is:

$$\frac{LAMBDA^S}{S!} \exp(-LAMBDA)$$

The Poisson distribution is extended to non-integer values of S using the relation between the cumulative distribution function of the Poisson distribution and the gamma distribution.

13.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next arguments is to be calculated.

Input Range: [1 : 3]

1. CUM and CCUM
2. S
3. LAMBDA

REAL (dpkind), OPTIONAL :: CUM. The CDF of the Poisson distribution.

Range: [0 : 1 - 10⁻¹⁰]

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the Poisson distribution.

Range: [10⁻¹⁰ : 1]

REAL (dpkind) :: S. The upper limit of summation of the Poisson density. The lower limit is 0.

Range: [0 : 10¹⁰⁰]

REAL (dpkind) :: LAMBDA. The mean of the Poisson distribution. Range:
[10^{-10} : 10^{100}]

INTEGER, INTENT(OUT), OPTIONAL :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 X outside range

-5 MEAN outside range

-6 SD outside range

3 CUM + CCUM is not nearly one

10 The cumulative Poisson is calculated as an incomplete gamma distribution.

This status indicates that an error occurred in the incomplete gamma code. It really shouldn't happen.

-50 Answer (if any) is below lower bound on range

50 Answer (if any) is above upper bound on range

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.

14 cdf_t_mod

```
SUBROUTINE CDF_T( WHICH, CUM, CCUM, T, DF, STATUS, CHECK_INPUT
)
REAL (dpkind) FUNCTION CUM_T(T, DF, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION CCUM_T(T, DF, STATUS, CHECK_INPUT)
REAL (dpkind) FUNCTION INV_T(CUM, CCUM, T, DF, STATUS, CHECK_INPUT)
```

14.1 The Distribution

The density is proportional to

$$\left[1 + \frac{T^2}{DF}\right]^{(DF+1)/2}$$

14.2 Arguments

INTEGER, INTENT(IN) :: WHICH. Integer indicating which of the next arguments is to be calculated.

Input Range: [1 : 3]

1. CUM and CCUM
2. T
3. DF

REAL (dpkind), OPTIONAL :: CUM. The CDF of the noncentral t distribution.

Range: $[10^{-10} : 1 - 10^{-10}]$

REAL (dpkind), OPTIONAL :: CCUM. One minus the CDF of the non-central t distribution.

Range: $[10^{-10} : 1 - 10^{-10}]$

REAL (dpkind) :: T. The upper limit of integration of the noncentral t density.

The lower limit is $-\infty$.

Range: $[-10^{100} : 10^{100}]$

REAL (dpkind) :: DF. The degrees of freedom.

Range: $[10^{-3} : 10^{10}]$

INTEGER, INTENT(OUT) :: STATUS. Return code.

-1 WHICH outside input range

-2 CUM outside range

-3 CCUM outside range

-4 T outside range

-5 DF outside range

3 CUM + CCUM is not nearly one

10 The cumulative t is computed as an incomplete beta distribution. This value indicates an error in the incomplete beta code. It really shouldn't happen.

-50 Answer (if any) is below the lower search bound

50 Answer (if any) is above the upper search bound

LOGICAL, INTENT(IN), OPTIONAL :: CHECK_INPUT. If PRESENT and .TRUE. input argument values are not checked for validity.